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Examination of the Variation in Students' Problem Solving Approaches Due to the Use of Mathematical Models in Doppler Effect

Öğrencilerin Doppler Etkisinde Matematiksel Model Kullanımına Bağlı Problem Çözme Yaklaşımlarındaki Değişimin İncelenmesi

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ABSTRACT: This study aims to investigate students' problem solving approaches by examining students' use of mathematical models. In this research, since many students think that the concepts of relativity are unfamiliar, abstract and difficult, it was focused on relativistic kinematics- the relativistic Doppler Effect. Sophomores from two cohorts of physics (n=60) and physics teaching (n=32) enrolled in a modern physics course at a university participated in the study. Participants were asked to provide extended written responses to the Doppler Effect problems with a test. Afterwards six students were purposefully selected for semi-structured interviews. Students' use of mathematical models revealed that students had difficulty in discriminating between fundamental concepts such as frequency and wavelength, source and observer, red-shift and blue-shift, and they consequently used these concepts interchangeably. In addition, because of students' the lack of ability of representing the problem in different forms according to a given physical context, they also had difficulty in determining the appropriate model. In conclusion, students used both physically and mathematically meaningless models and their problem solving approaches varied due to the use of mathematical models.

Keywords: Doppler Effect, problem solving, mathematical model, modern physics, physics education.

ÖZ: Bu çalışma, öğrencilerin matematiksel model kullanımını inceleyerek problem çözme yaklaşımlarını belirlemeyi amaçlamaktadır. Öğrencilerin çoğu görelilik kavramlarının alışılmadık, soyut ve zor olduğunu düşünmeleri sebebiyle, bu araştırmada göreli kinematiğe- göreli Doppler Etkisine- odaklanılmıştır. Araştırmaya bir üniversitenin ikinci sınıfında kayıtlı fizik (n=60) ve fizik eğitimi (n=32) olmak üzere modern fizik dersini alan iki grup öğrenci katılmıştır. Katılımcılardan testteki Doppler Etkisi problemlerine ayrıntılı olarak yazılı cevap vermeleri istenmiştir. Daha sonra yarı yapılandırılmış görüşmeler için altı öğrenci amaçsal örneklem ile seçilmiştir. Öğrencilerin matematiksel model kullanımları, frekans ve dalga boyu, kaynak ve gözlemci, kırmızıya kayma ve maviye kayma gibi bazı temel kavramları ayırt etmede zorluk yaşadıklarını ve bu kavramları birbiri yerine kullandıklarını ortaya çıkarmıştır. Ayrıca, öğrencilerin problemi verilen fiziksel bağlamda farklı formlarda ifade etme becerilerinin eksikliğinden dolayı öğrenciler uygun modeli belirlemekte de zorluk yaşamıştır. Bunun sonucunda öğrenciler hem fiziksel hem de matematiksel olarak anlamsız modeller kullanmış ve problem çözme yaklaşımları matematiksel model kullanımına göre değişkenlik göstermiştir.

Anahtar sözcükler: Doppler Etkisi, problem çözme, matematiksel model, modern fizik, fizik eğitimi.

1. INTRODUCTION

In our daily life, we experience the changes in the pitch of the sound of the siren when an ambulance is approaching or receding from us. The reason is one of the important phenomena of modern physics, the Doppler Effect, discovered in the 19th century by Christian Doppler. The Doppler Effect can be explained simply as the change in the frequency of the waves due to the motion of its source (Beiser, 2003). Although we mainly experience the Doppler Effect in sound waves in daily life, it is observed in both electromagnetic waves (light) and water waves.

The Doppler Effect for light is important in astronomy to determine the speed of astronomical objects while approaching and receding from each other. When an observer recedes from a light source, then red-shift occurs since the received frequency is lower than the source frequency, that is;

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$$f_{(\text{receding})} = f_0\sqrt{\frac{1-v/c}{1+v/c}} \quad (1.1)$$

and, when an observer is approaching the light source, blue-shift occurs since the received frequency is higher than the source frequency, that is;

$$f_{(\text{approaching})} = f_0\sqrt{\frac{1+v/c}{1-v/c}} \quad (1.2)$$

These two Doppler Effects for light are known as the longitudinal relativistic Doppler Effect in one-dimension (Beiser, 2003; French, 1968; Resnick, 1968). They can be derived directly by using the two postulates of the special theory of relativity (Moriconi, 2006). In addition, they can be applied to thought experiments to obtain the various phenomena of the relativistic kinematics such as "time dilation, length contraction, addition of velocities, Lorentz transformations, and the mass-energy relationship" in the special theory of relativity (Moriconi, 2006). Taking the challenges of learning about the concepts of the special theory of relativity into account, the extensive use of the Doppler Effect in many contexts may lead to alternative ways for teaching and learning the concepts related to the space, time, and reference frames.

With the introduction of the theory of relativity, many new concepts emerged. Pedagogical research about relativity can be classified as the studies of Galilean relativity (Bandyopadhyay, 2009; Hosson, Kermen, & Parizot, 2010; Panse, Ramadas, & Kumar, 1994; Ramadas, Barve, & Kumar, 1996) and the special theory of relativity. As it is seen in other domains of physics, the previous research has shown that students from high-school to post-graduate level had conceptual difficulties about concepts of the special theory of relativity (Dimitriadi, Halkia, & Skordoulis, 2005; Hewson, 1982; Hosson et al., 2010; Pietrocola & Zylbersztajn, 1999; Scherr, 2001; Scherr, Shaffer, & Vokos, 2001; 2002; Villani & Pacca, 1987). As the quantum theory, the special theory of relativity has changed the classical aspects of physics theories. Because of the importance of understanding of it, the special theory of relativity is included in high school curricula (Arriassecq & Greca, 2012). However, its highly abstract nature makes it even more difficult to learn and understand. For this reason, some research has focused on the development of new teaching approaches (Arriassecq & Greca, 2012; Villani & Arruda, 1998) and technologies (Barbier, Fleck, Perries, & Ray, 2005; Horwitz, Taylor, & Barowy, 1994) in order to overcome the difficulties in understanding the theory of relativity. Because of the importance of the role of teachers, recent studies have also focused on pre-service science/ physics teachers' conceptions about the special theory of relativity (Özcan, 2011; Sezgin Selçuk, 2011). In this study, we examined one of the important concepts of relativistic kinematics, the relativistic Doppler Effect (for light). The reasons for focusing on the Doppler Effect for light instead of classical Doppler Effect were that; (1) light is one of the fundamental concepts in modern physics and widely included in both relativity and quantum phenomena. So, the examination of the Doppler Effect for light rather than sound would be more appropriate for the current study, and (2) the relativistic Doppler Effect could reveal more about students' approaches to mathematical models because it includes the square root symbol that is having an important role in the statement of model. As a result, the research questions were:

- How do students understand the mathematical models of Doppler Effect?
- How does the use of a mathematical model shape students' problem solving approaches?

1.1. Models and Mathematical Modeling

A model can be defined as "a surrogate object, a conceptual representation of a real thing" (Hestenes, 1987); in other words, models are conceptual representations of physical systems and processes (Wells, Hestenes, & Swackhamer, 1995). In physics, many of the models are mathematical models since they work as the language of physics by expressing physical properties and statements of a theory in terms of equations (Greca & Moreira, 2002; Hestenes 1987). They are not mathematical elements only, because they carry the physical ideas and relationships within themselves (Bing & Redish, 2007). In physics classes, we widely witness that students memorize mathematical models of physical concepts (formulae) and try to solve physics problems without interpreting them, just in a plug and chug way. The reason might be that many students tend think that physics is a collection of statements of laws and formulae (Hammer & Elby, 2003), and that there is a strong relationship between physics and mathematics.

According to Hestenes (1987) there are four elements of a mathematical model: name, descriptive variable, equation of the model, and interpretation. Wells et al. (1995) asserted that in order to specify a mathematical model exactly, an interpretation is needed. Hestenes (1987) also stressed the importance of understanding and interpreting a model: "Students need to recognize the interpretation as a critical component of a model. Without an interpretation the equations of a model represent nothing; they are merely abstract relations among mathematical variables". It was found that the first year graduate physics students regarded mathematics as a mechanical method, and not as a way of constructive thinking (Breitenberger, 1992). Moreover, it was shown that students failed to interpret mathematical knowledge in physical contexts in spite of having relevant mathematical skills (Tuminaro & Redish, 2004). In another study (Steinberg, Wittmann, & Redish, 1996), students' failure in recognizing the relationships between actual physical situations and the associated mathematical model in mechanical waves were identified and found that, from the perspective of utilization of a mathematical model, novices and experts employed different knowledge organizations in problem solving. For example, although experts provided qualitative arguments, novices started to solve physics problems by using complex mathematical equations (Reif & Heller, 1982; Yiğit, Alev, Tural, & Bülbül, 2012). We also know that novice students attempt to solve physics problems just by assembling the individual equations in contrast to experts' successive refinement of the elements of problems (Larkin & Reif, 1979) and that novice students mostly memorize the formula and solve the problems in plug and chug way; when it does not work, they use another equation without thinking (Maloney, 1994). Another exploration (Dhillon, 1998) of expert and novice physics problem solving processes yielded that when there was not an obvious relationship among the elements of a problem, novices had difficulty in identifying the individual elements and used "symbols" to be able to make connections among the them. Similarly, when students' problem solving approaches are categorized (Walsh, Howard, & Bowe, 2007), it is seen that majority of students did not approach physics problems qualitatively in contrast to experts. These studies indicated students' inappropriate understanding of mathematical models and their physical meanings.

2. METHOD

In this study, we examined students' problem solving approaches in Doppler Effect. For this aim, we have focused on students' use of mathematical models for the relativistic Doppler Effect for "light". This study had two steps, namely the identification of students' problem solving paths in the first step, and testing these paths in the second step. The methodological procedures of the study are as follows:

2.1. Reliability and Validity Issues

The problems used in this research were examined by three independent experts majoring physics and physics education. The experts examined the problems using criteria including the appropriateness for grade level; appropriateness for the research aims (internal validity); verbal and mathematical appropriateness of the statements. In addition, internal reliability (dependability) (Fraenkel & Wallen, 2000) of the study was provided by getting the inter-coder reliability coefficient (0.86) that the researchers obtained by analyzing the data independently. Internal validity (credibility) of the study was provided by member checks, and the examination of test and interview questions by the experts.

2.2. Data Collection

2.2.1. Context

This research was conducted in an undergraduate level modern physics course. Modern physics is a one-semester compulsory course for all students majoring in physics and physics education. This course covers the following topics in the given order: special theory of relativity, introduction to quantum theory, properties of waves, the structure of atoms, the structure of nucleus, and radioactivity. This course had four class hours in a week and duration of each class was 50 minutes. The instructional approach in this course was mainly teacher-centered, enriched by instructional techniques such as analogy and role play. The of relativity concepts were taught during the first three weeks of the semester (almost 12 class hours). The Doppler Effect is one of the fundamental topics taught included in teaching of relativity concepts. After the Doppler Effect for sound is introduced, it is formulated for light with the explanation of red-shift, blue-shift, and transverse Doppler Effect concepts.

2.2.2. Participants

92 second-year students who were enrolled in a modern physics course at the physics department of a major research university in Turkey participated in this study. The test was implemented to all participants who were two cohorts majoring in physics teaching (n=32) and physics (n=60). In addition, six participants were selected for interview by purposive sampling. After the test was implemented, the students in each major problem solving category were identified; and by considering the representativeness of the categories and willingness of the students in each category, six participants for the interviews were determined.

2.2.3. Procedure

This study was conducted in two steps. In the first step, a test was implemented to 92 participants in order to obtain a pattern about students' problem solving approaches. In the test, students were asked two Doppler Effect problems. Both of the problems were examining the Doppler Effect for light in one-dimension; and one of them examined the blue-shift, the other one examined the red-shift. In the second step of the study, interviews were conducted with six students (three from each majoring in physics and physics education). In this step, three different Doppler Effect problems were asked to these students via semi-structured interviews. Two of these problems were again a one-dimensional Doppler Effect phenomenon for light, and the other question was examining the phenomenon in sound (classical) respectively. Interview protocols were provided to students to solve the problems and they were requested to think out loud as possible during qualitative thinking of the problems. Students were allowed to explain the elements of the mathematical models that they used. By this way, students provided an interpretation about the mathematical models they used. An interview per student took almost twenty minutes. All interviews were recorded by video camera.

2.3. Data Analysis

The data analysis was carried out starting from the first Doppler Effect problem in the test given in the first step of the study. The researchers started to conduct the analysis separately at first. By considering four elements determining the approach -starting point, use of frequency and wavelength concepts, identification of the observer and source, and concluding the problem-, six main categories were identified about students' problem solving paths. By excluding the sub categories under the main categories, the degree of agreement between the researchers was determined to be 0.86. Then, the second problem was examined according to these categories separately. After the discussion of all categories, full agreement about the main and sub categories was obtained.

Each step in determining the students' problem solving approaches was coded by a pathfinder constructed by the researchers that was specific for the Doppler Effect context. Table 1 presents the pathfinder which was used for the determination of students' problem solving approaches.

Table 1: The Pathfinder for Determining Students' Problem Solving

	CORRECT	WRONG (the use of red-shift and blue shift)	IRRELEVANT (the use of sound and transverse)	NOT
START	1	5	9	12
THE USE OF (ν - λ) and (ν_o - λ_o)	2	6	10	13
THE USE OF (ν - ν_o) and (λ - λ_o)	3	7	11	14
END	4	8		15

For example, ↓ might be the flow from 5 to 6, ↘ might be the flow from 1 to 6, 10 or 13, ↙ might be the flow from 12 to 2, 6 or 10).

In this table, each box represents a step in problem solving. For example, box number 1 represents the *correct start* to the problem solving by selecting the correct mathematical model of the Doppler Effect. Similarly, numbers 5 and 9 represent *wrong* and *irrelevant starts* to the problem solving, respectively. That means each box has both vertical and horizontal characteristics describing an element of problem solving approach. In addition, a dashed arrow indicates the probable flow among the elements composing the problem solving approach. More specifically, by using this pathfinder, statement of "1-2-3-4" means that the student started to solve problem correctly by selecting the correct model, next s/he used frequency or wavelength concepts correctly, then s/he used frequency or wavelength for observer and source correctly, and finally s/he got the correct result.

After the identification of students' problem solving paths in the first step, the categories were tested by means of the interviews with six participants in the second step. Each interview was assigned to an identified problem solving approach; however, since the total number of categories is greater than the number of interviewees, some of the sub categories could not be tested in the interviews. In the interviews, for each path, we also obtained some evidences about how students understood some physics concepts such as frequency, wavelength, blue-shift, red-shift etc., and how they used mathematical models to reach a conclusion in solving the Doppler Effect problems.

3. FINDINGS

The data obtained from 92 students indicated a variation in students' problem solving approaches in Doppler Effect with six categories. Although some of these paths were probable, most of them were context-specific and providing extensive information about students' understanding of fundamental concepts in modern physics. These paths were: (1) Correct path, (2) Missing minus/plus signs, (3) Wrong cancels another wrong, (4) Shortcut without physical interpretation, (5) Conceptual or/and mathematical difficulty, and (6) Not making sense the other contexts by using the irrelevant models.

In order to keep the focus, the current study presents the categories and examples for each category by focusing one of the questions. One-dimensional examination of the Doppler Effect by the revision of the famous anonymous Doppler Effect problem for light was given below.

Problem: At what speed would a motorist in a very fast car have to go so that he would see a red traffic light as green? Assume that the light looks red when the motorist is at rest. (Use λred =650 nm, and $\lambda green$ =530 nm)

3.1. Path 1: Correct Path (from correct start to correct conclusion)

The students in this category have reached the correct conclusion by following the right steps (boxes 1-2-3-4 in Table 1). This path showed that students knew when and how they should use the mathematical models. For this reason, they have chosen the correct blue-shift or red-shift formula before starting to solve problem by correctly interpreting the different situations about the Doppler Effect. Lastly, students did not make any mathematical mistakes while carrying out calculations. Appendix 5.1 shows an example for the correct path. As it is seen in Appendix 5.1, the student selected the correct mathematical model (blue-shift). The student discriminated wavelength and frequency, and used observer and source wavelength concepts correctly. Without any mathematical mistake, he reached the correct conclusion. This path is the expected path by physics instructors. However, in this case, test results revealed that almost 10% of students have followed this path. Another point we observed in this group was that they use some visual representations to make sense out of the questions, while solving the problems. Similarly, in the interviews also, most of the students started to solve the problems by using visual representations, they determined receding or approaching behavior of the source, and then they selected the mathematical model (blue-shift or red-shift). It also showed that these students could use multiple representations by transferring information into different formats.

3.2. Path 2: Missing Minus/Plus Signs (from wrong start to correct conclusion)

The students in this category, started to solve this problem wrongly by red-shift formula instead of the blue-shift. At first, the students constructed the mathematical model by using wavelength. Definitely, it is possible to have the blue-shift formula by using wavelength rather than frequency. However, while manipulating a mathematical model, the elements and signs composing the model should be carefully analyzed and used. In this approach, the students ignored to change the signs inside the square root while constructing the model by using wavelength. Thus, although the square root part of the model looks like the blue-shift formula, the model indicated the red-shift. Therefore, they started to solve the problem wrongly by using the signs wrongly. These students used wavelength and frequency correctly in the model, and they could identify the observer and source values correctly. During the solution of the problem, students encountered a problem. That was the requirement of "minus sign". The students in this category simply ignored the minus sign at the end of the problem, and by this way they reached the correct conclusion. That means that although they started to solve question wrongly (boxes 5-2-3-4 in Table 1), by ignoring the signs, they got the correct result for the problem. Appendix 5.2 shows a sample from this type of approach to the Doppler Effect problem. In Appendix 5.2, student's solution shows clearly ignorance of the signs. The student started to solve the problem by construction of the model wrongly, however reached the correct conclusion by ignoring the "minus sign" for velocity. Students might do that unconsciously (i.e. not recognizing the sign changes), or they might do that consciously because of the concepts of "speed, speed of light etc.". For this reason, the students might recognize that speed should not have minus sign, and then they might ignore the minus sign in order to reach the correct conclusion. No matter how students use this approach and get a correct result, it is neither mathematical nor physical.

3.3. Path 3: Wrong Cancels another Wrong= Correct Conclusion!

One of the interesting results comes from the students who follow this path. These students obtained the correct conclusion by chance. That was due to the cancellation of two mistakes each other and converting the result to correct (boxes 5-6-3-4 in Table 1). In this path, students took two wrong steps at the beginning, and these wrongs cancelled of each other, and the students reached the correct end. At first, students could not discriminate blue-shift and red-shift, so they started to solve problem with wrong selection of mathematical model. Then they continued to use wavelength and frequency interchangeably. At the end, they reached the correct conclusion. This was due to the nature of mathematical models of blue-shift and red-shift situations. However, while solving the problem, students did not seem to be aware of using both of these concepts interchangeably. They were also not aware of the mathematical structure of the formula leading such type of conclusion. This mathematical approach of the students showed their lack of qualitative interpretation of the symbols and concepts. As it is presented in Appendix 5.3, the student started to solve question wrongly by constructing the red-shift formula. Then, the student continued with the second wrong that was the use of wavelength and frequency interchangeably. The student set the observer and source values correctly. At the end of the mathematical calculations, the student reached the correct conclusion.

3.4. Path 4: Shortcut without Physical Interpretation

Some of the students followed a shorten path to reach the conclusion (boxes 1-13-14-4 in Table 1). The students in this group identified blue-shift and red-shift situations correctly, and then they started solution in the right way. After the determination of the correct mathematical model, they identified the values for both observer and source wavelengths and frequencies fast, so they got a shortcut formula to get the correct conclusion. Appendix 5.4 and Appendix 5.5 show the different uses of the shortcuts. Appendix 5.4 presents the students' correct interpretation by stating "observer approaching to the source" and correct selection of mathematical model of Doppler Effect. By using shortcut, he got the correct result. Appendix 5.5 also presents a shortcut approach. Since the problem stated the values of the wavelength, the student wrote the correct model at the beginning and proceeded very fast. He also used the signs correctly in the square root part of the model. By this way, he reached the conclusion immediately.

3.5. Path 5: Conceptual or/and Mathematical Difficulty

3.5.1. Path 5a: Difficulty in computation (from correct start to no conclusion)

The students, who followed this path, started to solve problem correctly. They understood the situation and selected the mathematical model correctly by discriminating blue-shift and red-shift. They could also discriminate wavelength and frequency, and put the observed wavelength and source wavelength correctly in the Doppler Effect formula. The students following this path did not present a conceptual difficulty; however, the difficulty in mathematical computations prevented students arriving at the correct conclusion (boxes 1-2-3-15 in Table 1). An example for this pattern is presented in Appendix 5.6, this student used all concepts and related mathematical model correctly. Although, he did not display a conceptual problem about the Doppler Effect, he could not reach the correct conclusion. This shows that some of the students got lost in the mathematics while doing physics. If the students could handle mathematical calculations, they would have reached the correct conclusion as in the first path. However, insufficient mathematical background caused difficulty in computation and limited those students getting correct results.

3.5.2. Path 5b: Conceptual difficulty is first (from wrong start to no conclusion)

The students, who use this path, presented some clues of both conceptual and mathematical difficulties (boxes 5-2-3-15 in Table 1). They started to solve the problem wrongly by using the red-shift formula. Although they used wavelength and frequency, and observer and source concepts correctly, they got lost in the mathematical calculations. As the results of conceptual and mathematical mistakes, they could not reach the correct conclusion. This path indicated where the problems emerged; both from mathematics and physics. Appendix 5.7 presents a sample for this category. As it is seen in Appendix 5.7, the student started to solve the problem wrongly because of conceptual difficulty in understanding of the Doppler Effect. Finally, the student could not reach the correct conclusion because of the mathematical difficulty.

3.5.3. Path 5c: Mathematical difficulty is dominant (from correct start to no conclusion)

The students following this path started to solve the problem correct by discriminating blue-shift and red-shift correctly. They could also identify the variables related to the observer and the source correctly (boxes 1-6-3-15 in Table 1), however, they were not able to distinguish the wavelength and frequency from each other. Again, due to both conceptual and mathematical mistakes, these students could not reach the correct conclusion. Appendix 5.8 presents an example for this category. As it is seen in Appendix 5.8, the student has both conceptual and mathematical problems. Although the student started to solve the problem correctly, some conceptual and mathematical problems prevented her arriving at the correct conclusion. The difference between this group and the first group in this category is that the students following Path 5a did not make a conceptual mistake; however, in this group, not getting a conclusion is the result of both conceptual and mathematical difficulties of students.

3.6. Path 6: Use of Irrelevant Models

The number of students who use irrelevant models were quite large. The use of irrelevant models presented some evidences about students' conceptual and also mathematical difficulties about the Doppler Effect. This path has two sub-categories.

3.6.1. Path 6a: Doppler Effect has a unique model! (from irrelevant start to wrong conclusion)

In the previous paths, we observed some examples on students' inability to discriminate red-shift and blue-shift by poor conceptual understanding of the Doppler Effect. In this case, we see that, some students could not discriminate the models of sound and the transverse model of the Doppler Effect for light (boxes 9-10-11-8 in Table 1). Due to lack of conceptual understanding of the concepts, they started solving the problem by constructing irrelevant models because they could not interpret the context and related concepts. Appendix 5.9 and Appendix 5.10 present the examples for sound and the transverse model of the Doppler Effect for light, respectively. In the appendices, students' inadequate conceptual understanding of the context and lack of qualitative inquiry can be observed at the beginning of the problem, although they got a result at the end. It is because students were not aware of why they used that model while solving the problems. In addition, as we see the problem solutions in Appendix 5.9 and Appendix 5.10, students drew a box around the value after they got the conclusion. This can be interpreted as getting a result is more important than how it is obtained. As a result, although students arrived at a conclusion, this approach is neither physically meaningful for physics contexts nor an expected outcome of physics classes.

3.6.2. Path 6b: Doppler Effect has a unique model! (from irrelevant start to no conclusion)

The only difference of this path from the previous path is that the students in this group could not get a conclusion (boxes 9-10-11-15 in Table 1). This indicates that the students in this group had difficulty in both making sense of the Doppler Effect phenomenon and the mathematical computations emerging during problem solving. As it is seen in Appendix 5.11, the student used an irrelevant model to solve the Doppler Effect problem. Then, the student did not complete the problem solving steps and could not reach a conclusion. This paths also gives the clues about these students have both conceptual and mathematical difficulties. In addition, we can observe that the levels of conceptual difficulty of students differ. That means, in the previous examples, we observed some students had problems about discriminating red-shift and blue-shift, or frequency and wavelength, or source and observer; however, in this case students had problems in making sense the physical situation at the beginning and they used irrelevant models.

3.7. No Answer

Small number of students could not have an answer for the Doppler Effect problems. Boxes 12-13-14-15 in Table 1 present that students did not provide a solution for the problem.

4. CONCLUSION and DISCUSSION

In this study, we examined students' problem solving approaches by focusing on students' use of mathematical models in Doppler Effect. The results showed that students displayed some conceptual and mathematical difficulties. The difficulties for each problem solving path can be summarized as in Table 2.

Table 2: Students' Conceptual or Mathematical Difficulty in Each Problem Solving Path

Paths	Conceptual Difficulty	Mathematical Difficulty
Path 1: Correct path	-	-
Path 2: Missing minus/plus signs	+ (*)	+
Path 3: Wrong cancels another Wrong = correct conclusion!	+ (*,**)	-
Path 4: Shortcut without physical interpretation	-	-
Path 5a: Difficulty in computation	-	+
Path 5b: Conceptual difficulty is first	+ (*)	+
Path 5c: Mathematical difficulty is dominant	+ (**)	+
Path 6a: Doppler Effect has a unique model!	+ (*,**,***)	-
Path 6b: Doppler Effect has a unique model!	+ (*,**,***)	+

The signs mean that;(-): no difficulty, (+): there is a difficulty. Types of difficulty; (*****): difficulty in understanding red-shift and blue-shift concepts, (******): difficulty in understanding of ν or λ, (*******): difficulty in understanding of observer' and source' ν or λ.

Hestenes (1987) and Wells et al. (1995) indicated that interpretation of a mathematical model in physics was very important. In addition, they explained that mathematical models were just mathematical rather than physical without physical interpretation. For this reason, as it is seen in Table 2, students' inability of interpreting the elements of mathematical model and the model itself presented some information that students had some conceptual difficulty about the important concepts of modern physics. These were: difficulties in understanding red-shift and blue-shift concepts, difficulty in understanding of frequency or wavelength, and problem in understanding of observer' and source' frequency or wavelength. For this reason, they mainly used the concepts like frequency and wavelength interchangeably. In addition, they had difficulty in identifying of the Doppler Effect for different wave sources in two-dimensions.

The previous research on students' approaches to solving physics problems and use of mathematics revealed that students solve the problems by lacking a qualitative thinking and they use mathematics just as in plug and chug way (Breitenberger, 1992; Dhillon, 1998; Larkin & Reif, 1979; Maloney, 1994; Reif & Heller, 1982; Steinberg et al., 1996; Tuminaro & Redish, 2004; Walsh et al., 2007). Similar to these studies, in this study we also saw that the majority of students did not approach solving problems in a scientific and strategic manner with a qualitative interpretation. Many students began to problem solving by not rendering the physical meaning of the mathematical formulations and calculations. Mathematics is an important tool, which shapes the formalism of the physical theories and it explains the statements of the theory in terms of equations. In other words, the mathematical symbols are the set of statements of the physical theories without using their semantic content. Of course, mathematical operations are important; however, it makes no sense, if the mathematical representations of the variables are not used with its semantic interpretations gathered through the models. So, it is meaningless to solve a physics problem without interpreting the physical event of required mathematical model. Although physics uses mathematics, it is different from the mathematics. Interpretation of a mathematical model discriminates a physicist and mathematician (Bing & Redish, 2007; Redish & Gupta, 2009). Moving directly to the solution without questioning the physical context, which is the problem based on, makes incorrect conclusions inevitable. The physical context and the mathematical model to be used should be paired with, and the blending (Bing & Redish, 2007) that consists of these two variables should be put to work in the mental process. In the problems with a predictable solution or requiring simple mathematical operations, physical context was not be much dwelled upon. In such cases, it was observed that no image was formed related to the physical context in the minds of the students (Bing & Redish, 2007).

In the problems related to the Doppler Effect, which were used in this study, placing the variables in the formula without dealing with the physical context or trying to reach the solution by trial and error brings the mistakes. Students focusing on just getting a result disregarded the minus sign related to frequency and wavelength in order to reach the correct conclusion. That means, the reason of such type of a mistake can be explained by using any mathematical model directly without making a qualitative analysis at first. However, if the problems had been analyzed qualitatively and the reference frame concepts (driver and traffic light) had been considered together in the construction of mathematical model, then the students might have been noticed where the minus sign stemmed from. Some of them modified the mathematical calculations in accord with their expectations. That means knowledge of some concepts caused students to modify their calculations by ignoring plus/minus signs etc. No matter this is done by consciously or unconsciously, it indicates students ignored how a change in the mathematical formula changes the physical meaning. While some students tried to overcome mathematical calculations by ignoring the signs, in some of the cases, students got lost in the equations and could not reach the intended solutions. In addition to the calculation errors during problem solving, wrong physical interpretations also made the problems more complex, so they might not overcome the basic algebra to solve the problems. These results are compatible with the findings of Larkin and Reif (1979), Maloney (1994), Dhillon (1998) and Walsh, Howard and Bowe (2007) indicating novice problem solvers' inappropriate use of mathematical elements and symbols without thinking properly during problem solving in contrast to experts.

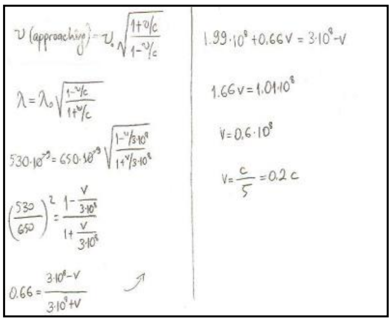
In another case, by using wavelength instead of frequency in the Doppler Effect formula, the solution was designed as red-shift rather than blue-shift. Although the students started solving the problem incorrectly, they reached the correct conclusion. The underlying reason for the incorrect selection of the blue-shift and the red-shift might be again the inability to determine the reference frames correctly. Reference frame is important for relative motion (Scherr et al., 2001). Since relative motion of the source and observer has importance for the Doppler Effect of light, understanding of reference frame is interrelated with understanding of the Doppler Effect. It is necessary to construct a mathematical model according to the observer in the car that approaches the traffic lights. However, because a mathematical model needed for the physical situation was not determined, students started to solve problem by using the wrong models. In one of the cases, we see that some students got the correct result although they used wavelength and frequency incorrectly twice. The variables such as frequency and wavelength-which are the main elements in the Doppler Effect formula- were used interchangeably by some of the students. Some other cases showed that students reached the correct conclusion, quite accidentally, due to against confusion about the wavelength and frequency. In other words, some of these students took shortcuts from the first step to the forth.

These findings were significant as they cannot be detected by multiple-choice tests. If these were multiple-choice type test questions, students could have chosen the correct answers with interchangeably using the basic concepts, ignoring the signs, or wrong qualitative interpretation. For this reason, this type of examination provided more information about students' conceptions as well as problem solving approaches and it discriminated students' inappropriate paths during problem solving. Furthermore, this finding puts forth the importance of constructing or determining the appropriate mathematical model for the physical context. If the mathematical model and the physical context are used compatibly by qualitative inquiry during the problem solving, correct solution might be more probable than the wrong one. One of the important goals of physics courses is to help students to solve the problems, which they encountered in everyday life by transferring their knowledge and understanding to real world situations. As Redish, Scherr and Tuminaro (2006) claimed, by means of the results of individuals' problem solving, individuals could understand and reconsider their intuitions about the physical world better. We should take care of students' solving physics problems and examine what students cannot do as well as what they can do. Students' conceptual and mathematical difficulty might be determined with the new strategies and students' problem solving might be facilitated by relating the mathematical model with the physical phenomena. In this study, it was not aimed to generalize the problem solving paths for all Doppler Effect problems; however, from this point forth it is expected that this study could be a model for the future pedagogical research on the Doppler Effect.

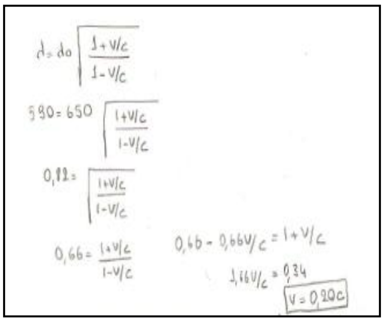
5. APPENDICES

The samples for students' problem solving approaches.

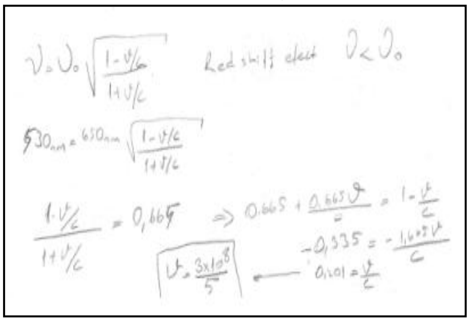
Appendix 5.1. An example for Path 1 (Correct path)



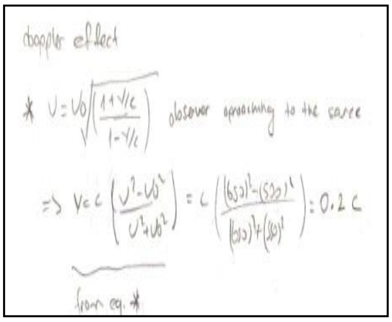
Appendix 5.2. The example for Path 2 (Missing minus/plus signs)



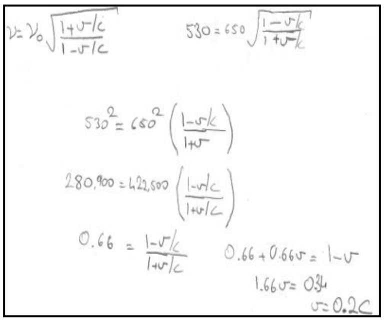
Appendix 5.3. The example for Path 3 (Wrong cancels another wrong= Correct conclusion)



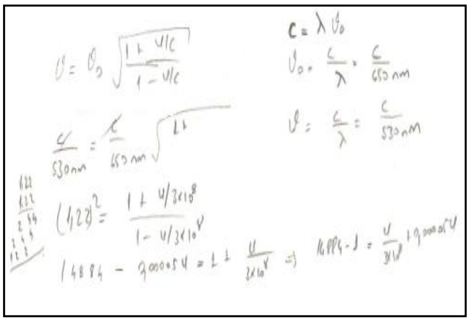
Appendix 5.4. An example for Path 4 (Shortcut)



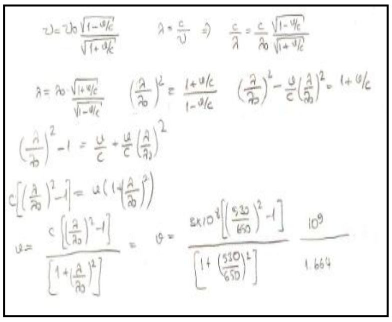
Appendix 5.5. Another example for Path 4 (Shortcut)



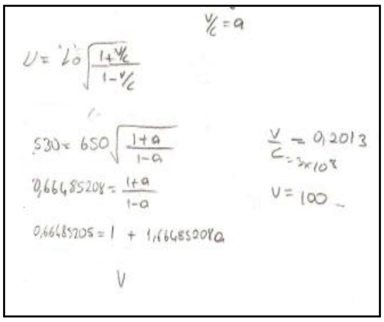
Appendix 5.6. The example for Path 5a (Difficulty in computation)



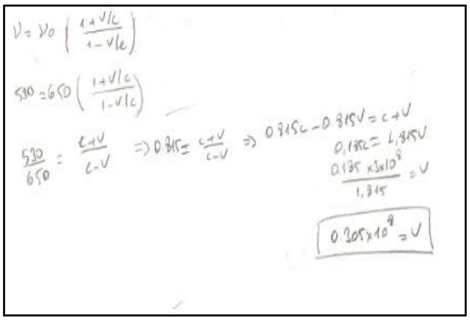
Appendix 5.7. The example for Path 5b (Conceptual difficulty is first)



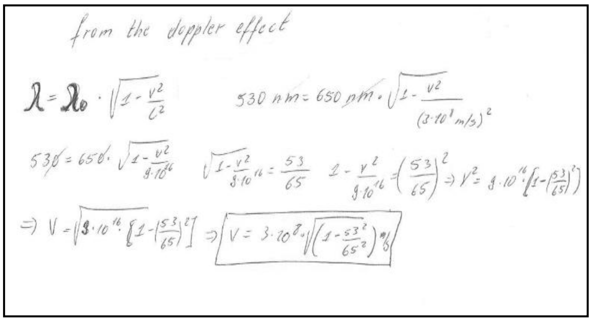
Appendix 5.8. The example for Path 5c (Mathematical difficulty is dominant)



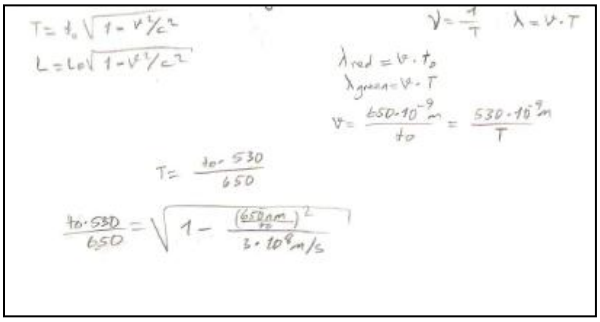
Appendix 5.9. The example for Path 6a for sound (Doppler Effect has a unique model!)



Appendix 5.10. The example for Path 6a for transverse (Doppler Effect has a unique model!)



Appendix 5.11. The example for Path 6b (Doppler Effect has a unique model!)



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Uzun Özet

Modern fizikteki en önemli olgulardan birisi olan Doppler Etkisi 19. yüzyılda Christian Doppler tarafından keşfedilmiştir. Doppler Etkisi kısaca dalgaların frekansının kaynağının hareketine bağlı olarak değişimi şeklinde açıklanabilir (Beiser, 2003). Doppler Etkisini günlük hayatta ses dalgalarında görmemize rağmen, elektromanyetik dalgalar (ışık) ve su dalgalarında da Doppler Etkisi gözlenir. Literatür, öğrencilerin liseden yüksek lisans seviyesine kadar özel görelilik teorisi kavramlarında kavramsal zorlukları olduğunu ortaya koymuştur (Dimitriadi, Halkia, & Skordoulis, 2005; Hewson, 1982; Hosson, Kermen, & Parizot, 2010; Pietrocola & Zylbersztajn, 1999; Scherr, 2001; Scherr, Shaffer, & Vokos, 2001; 2002; Villani & Pacca, 1987). Konuların bir hayli soyut olması öğrenmeyi zorlaştırmaktadır. Bu sebeple zorlukları gidermek ve öğrenmeyi kolaylaştırmak için araştırmacılar yeni öğretim tekniklerinin (Arriassecq & Greca, 2012; Villani & Arruda, 1998) ve teknolojilerinin geliştirilmesine odaklanmıştır (Barbier, Fleck, Perries, & Ray, 2005; Horwitz, Taylor, & Barowy, 1994). Öğretmenlerin rolünün önemi ile son zamanlardaki çalışmalar öğretmenlerin ve öğretmenlerin özel görelilik teorisini nasıl anladıklarına odaklanmıştır (Özcan, 2011; Sezgin Selçuk, 2011). Modeller fiziksel özellikleri ve teorik ifadeleri denklemlerle ifade ederek fiziğin dili gibi işlev yaptıklarından, fizikte bir çok model matematiksel modeldir (Greca & Moreira, 2002; Hestenes 1987). Fakat Tuminaro ve Redish (2004) öğrencilerin yeterli matematiksel beceriye sahip olsalar da fizik bağlamlarında matematiksel bilgiyi yorumlamada başarısız olduklarını ortaya koymuştur. Ayrıca çalışmalar öğrencilerin matematiksel modelleri ve fiziksel anlamlarını isabetsiz anlamalarını işaret etmektedir (Larkin & Reif, 1979; Maloney, 1994; Tuminaro & Redish, 2004; Reif & Heller, 1982; Walsh, Howard, & Bowe, 2007). Sonuç olarak, literatürdeki araştırmalar öğrencilerin çoğu görelilik kavramlarının alışılmadık, soyut ve zor olduğunu düşünmelerini göstermektedir. Bu sebeple araştırmada öğrencilerin doğru modellere nasıl karar verdiklerini belirlemek için farklı bağlamlarda Doppler Etkisine odaklanıldı. Diğer bir deyişle, bu çalışma, öğrencilerin matematiksel model kullanımını inceleyerek problem çözme yaklaşımlarını belirlemeyi amaçlamaktadır. Böylece araştırma soruları şunlardır: (1) Öğrenciler Doppler Etkisinin matematiksel modellerini nasıl anlamaktadırlar? (2) Matematiksel model kullanımı öğrencilerin problem çözme yaklaşımlarını nasıl şekillendirir? Araştırmaya bir üniversitenin ikinci sınıfında kayıtlı fizik (n=60) ve fizik eğitimi (n=32) olmak üzere modern fizik dersini alan iki grup öğrenci gönüllü olarak katılmıştır. Modern fizik tüm fizik ve fizik eğitimi öğrencileri için alınması zorunlu tek dönemlik bir derstir. Bu ders özel görelilik ve kuantum teorilerinin konularını kapsar. Görelilik kavramları dönemin başladığından itibaren ilk üç haftada öğretilir (yaklaşık 12 ders saati). Doppler Etkisi de görelilik kavramlarının öğretiminde temel kavramlardan biridir. Ses için Doppler Etkisi kavramının öğretiminden sonra, kırmızıya kayma, maviye kayma ve enine Doppler Etkisi kavramları ile ışık için formülize edilir. Bu çalışma iki basamakta gerçekleştirilmiştir. Birinci basamakta öğrencilerin problem çözme yaklaşımlarına ilişkin yapı elde edebilmek için 92 öğrenciye test uygulanmıştır. Test bir boyutta kırmızıya kayma ve maviye kayma olmak üzere iki soru içermektedir. Katılımcılardan Doppler Etkisi problemlerine ayrıntılı olarak yazılı cevap vermeleri istenmiştir. Çalışmanın ikinci basamağında yarı yapılandırılmış görüşmeler için altı öğrenci (fizik ve fizik eğitimi öğrencilerinden üçer kişi) amaçsal örneklem ile seçilmiştir. Bu basamakta öğrencilere üç farklı Doppler Etkisi problemi sorulmuştur. Öğrencilerin problem çözme yaklaşımının belirlenmesinin her basamağı araştırmacılar tarafından Doppler Etkisi bağlamına hazırlanan kılavuz ile kodlanmıştır. Tablo 1 bu kılavuzu göstermektedir. Bu tabloda her bir kutu problem çözümünde bir basamağı temsil eder. Mesela, 1 numaralı kutu Doppler Etkisinde doğru matematiksel modeli seçerek problem çözmeye doğru başlamaya karşılık gelir. Benzer şekilde 5 ve 9. numaralar problem çözmeye yanlış ve ilgisiz başlamayı temsil eder. Bu kılavuzun kullanımında, mesela, "1-2-3-4" öğrencinin problem çözmeye doğru modeli seçerek doğru başladığını, sonra frekans veya dalgaboyu kavramlarını doğru kullandığını, daha sonra gözlemci ve kaynak için frekans veya dalgaboyunu doğru kullandığını ve son olarak doğru sonuca ulaştığını ifade eder. 92 öğrenciden elde edilen sonuçlar öğrencilerin Doppler Etkisinde problem çözme yaklaşımında 6 kategoriden oluşan bir varyasyonu işaret etmektedir. Bazı yaklaşımlar muhtemel olsa da bir çoğu bağlama özel ve öğrencilerin modern fizikte bazı temel kavramları anlamaları hakkında kapsamlı bilgi vermektedir. Bu yaklaşımlar: (1) Doğru yol, (2) Eksi/artı işaretinin ihmali, (3) Yanlış diğer yanlışı yok eder, (4) Fiziksel yorumdan yoksun kısa yol, (5) Kavramsal ve/ya matematiksel zorluk, ve (6) İlgisiz model kullanımı ile diğer bağlamları anlamlandıramama. Öğrencilerin matematiksel model kullanımları öğrencilerin frekans ve dalga boyu, kaynak ve gözlemci, kırmızıya kayma ve maviye kayma gibi bazı temel kavramları ayırt etmede zorluk yaşadığını ve dolayısıyla onları birbiri yerine kullandığını ortaya çıkarmıştır. Ayrıca, öğrencilerin problemi verilen fiziksel bağlamda farklı formlarda ifade etme becerilerinin eksikliğinden dolayı öğrenciler uygun modeli belirlemekte de zorluk yaşamıştır. Bunun sonucunda öğrenciler hem fiziksel hem de matematiksel olarak anlamsız modeller kullanmış ve problem çözme yaklaşımları matematiksel model kullanımına göre değişkenlik göstermiştir. Bu bulguların çoktan seçmeli testlerle belirlenmeyeceği manidardır. Eğer çoktan seçmeli testler kullanılmış olsaydı, öğrenciler temel kavramları birbiri yerine kullanarak, işaretleri ihmal ederek ve yanlış nitel yorumlamalar ile doğru cevapları seçebilecekti. Bu sebeple bu tip bir inceleme öğrencilerin problem çözme yaklaşımlarının yanında kavramsal bilgilerini de ortaya çıkartmış ve öğrencilerin problem çözmede uygun olmayan yollarını ayırt etmiştir. Öğrencilerin matematiksel modelleri kullanımlarının incelenmesi yaklaşımı ile farklı fizik konularında yaşadıkları matematiksel ve fiziksel zorluklar ortaya çıkarılabilir ve bu zorlukların giderilmesi ile ilgili daha spesifik çözümler elde edilebilir.

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